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TM-1036  
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A TABULATION OF  
A FEW THEORETICAL CALCULATIONS  
FOR SATELLITE REFRIGERATOR EXPANDERS

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May 8, 1981

## INTRODUCTION

Figure 4-4 of the Doubler Design Report (May, 1979) illustrates the typical configuration of satellite refrigerator heat exchangers and expanders. There are two reciprocating expansion engines in each satellite: a "Gas" or "Dry" engine and a "Wet" or "Liquid" engine.

Two kinds of Dry engines are currently used in satellite refrigerators: the Gardner-Fermi Dry and the CTI Dry. The Gardner-Fermi Dry is now essentially an in-house design resulting from several years of attempts by many people here at the laboratory to bring the engines purchased from Gardner Cryogenics to an acceptable level of reliability. Almost none of the original Gardner details remain, but the original form is still there - a single stainless steel piston in a stainless steel cylinder with "cold" graphite-teflon piston rings, a "warm" piston shaft seal, pushrods operating "cold" rocker arms which allow valves to be placed in the cylinder head directly under the piston, all-metal valves, and a flywheel cantilevered off the crankshaft.

The CTI Dry and Wet engines are basically Model 1400 refrigerator expanders made by Helix Process Systems in Waltham, Massachusetts. The expanders consist of two phenolic pistons closely fitting in stainless-steel cylinders with no "cold" piston rings, only O-rings on the top aluminum portion of the piston at the entry to the cylinder.

The Dry has two three-inch diameter pistons, the Wet two, two-inch diameter pistons; otherwise the Dry and Wet are identical. The valves are operated by pullrods and are located to the side of the cylinder head, valve rocker arms are not used. A flywheel is centered between the two cylinders. The two pistons operate  $180^{\circ}$  out of phase so each has its own cams and valves and the "engine" is essentially two single-piston expanders operating in parallel and connected to the same crankshaft and fly wheel, The CTI Wet engine will be the standard satellite refrigerator wet engine.

This table of calculated results (figure 1) is an attempt to summarize some basic aspects of the theoretical performance of these expanders. A separate TM will summarize actual efficiencies calculated from data taken from operating Dry expanders during 1980.

#### EXPLANATION OF ENGINE TABLE CALCULATIONS

##### Columns 2 and 3

The intake cutoff is the ratio of intake volume to total volume swept out by the piston. It can be found from  $\text{cutoff} = (1 - \cos \theta) / 2$  where  $\theta$  is the angle, measured from the point where cylinder volume is a minimum, at which the intake valve closes. If the timing is set such that the intake valve opens at this minimum-volume point then  $\theta$  is just the intake cam dwell angle.

In the case of the Gardner-Fermi engine the cam follower operates the valve pushrod by means of a tappet bolt. The clearance between this tappet bolt and the pushrod cap is adjustable. For 0.005 inch. tappet clearance the cam (figure 2) must turn  $6^{\circ}$  to make contact of the tappet on the pushrod cap and  $6^{\circ}$  again after breaking contact. Hence at least  $12^{\circ}$  of cam angle is "lost" in making up this tappet clearance. Actually the record of cylinder pressure indicates that at least  $15^{\circ} - 20^{\circ}$  of cam angle is "lost", perhaps  $3^{\circ}$  or more going to backlash, compression of the pushrod,

the restriction of flow through the partially open valve, etc. The cams on the Gardner-Fermi engine are advanced  $8^{\circ}$  so that with 0.005 tappet clearance the intake valve opens at approximately the point where cylinder volume is minimized.

Note that as tappet clearance increases due to wearing of the tappet bolts or pushrod caps the valves open later and close earlier. When the intake valve closes earlier the intake cutoff is decreased. Figure 3 summarizes the effect of tappet clearance on intake cutoff for constant cam position. Although the  $100^{\circ}$  cam (figure 3) is standard, a 0.38 cutoff is probably typical.

For the CTI dry engine pullrod collet clearance, like the tappets of a Gardner-Fermi engine, determines the amount of cam used. Using the plot of cam follower position versus cam rotation (figure 4), I have estimated cutoffs based on cam position and collet clearance (figure 5).

It has been observed that collet clearance shrinks by about 0.012 inches upon cooling the engine, the pullrod apparently shrinking more than the valve body. A CTI engineer told me that during a cold test of an engine like our dry engines (but not one of ours) a warm collet clearance of at least 0.020 inches on an exhaust valve pullrod completely disappeared, causing the exhaust valve not to close completely. This he attributed to the 0.012 shrinkage of the collet clearance plus lifting of the crankshaft-cam assembly by the loaded piston. This indicates that collet clearances during operation may depend on several factors and may be different for different engines.

For the CTI wet engine the inlet cam (figure 6) has a steep drop which makes a 0.010 and 0.020 collet clearance provide a 0.66 and 0.65 intake cutoff, respectively.

The effects of tappet clearance and collet clearance on engine performance may be greater than these calculations indicate since the amount that the valve opens is reduced by the amount of tappet or collet clearance. In the case of the CTI dry engine the cam follower lift is only 0.080 inches, so increasing collet clearance from 0.010 to 0.030 reduces valve travel from 0.070 to 0.050 inches. If flow area is proportional to valve travel this doubles the pressure drop through the valve

#### Column 5

Cylinder pressure after expansion (but before the exhaust valve opens) was found from  $P = P(\rho, s)$  where  $s$  = entropy is assumed constant and  $\rho$  = density is found from  $\rho_{\text{expanded}} = \rho_{\text{inlet}} \times (\text{intake cutoff})$ .

Dry engines: The Design Report (1979) inlet conditions are  $30^\circ\text{K}$ , 20 atm. Hence  $\rho_{\text{inlet}} = 3.118 \times 10^{-2} \text{ g/cm}^3$  and  $s = 13.03 \text{ J/g}^\circ\text{K}$ .

For example, for the cutoff = 0.46,  $\rho_{\text{expanded}} = (3.118 \times 10^{-2} \text{ g/cm}^3) \times (0.46) = 1.434 \times 10^{-2} \text{ g/cm}^3$ . The entry in the NBS helium tables closest to this density and  $s = 13.03 \text{ J/g}^\circ\text{K}$  is  $P = 5 \text{ atm}$ .

Wet engine: The same procedure was followed for three different temperatures and 20 atm with the pressure found from the NBS T-S diagram for helium in the last two cases.

In general the final cylinder pressure will be increased by inefficiencies which increase gas temperature (or entropy) and decreased by ring blowby or exhaust valve leakage. Clearly a leaking exhaust valve could significantly reduce cylinder pressure, but the pressure increase (relative to ideal) due to inefficiency may also be large.

For the wet engine with 8°K inlet gas and an 80% efficient expansion the cylinder pressure after expansion is about 4.0 atm instead of the 2.5 atm in the table. This was found by first estimating the ideal change in enthalpy, then finding the enthalpy resulting from a change 80% of ideal, and finally finding  $P_{out} = P(h, \rho)$  where  $h$  is this new enthalpy and  $\rho$  is the same final density.

When the exhaust valve opens the gas is exhausted to the exhaust side pressure, generally 1.2 atm for the dry engine and 1.8 atm for the wet engine. This final pressure drop (or rise) is assumed to be isenthalpic extracting no energy from the gas and changing gas temperature like a J-T valve would.

#### Column 6

Enthalpy after expansion is found from  $h = h(\rho, s)$  where  $s$  = entropy is assumed constant and  $\rho$  = density is found from  $\rho_{expanded} = \rho_{inlet}^x$  (intake cutoff).

Dry engines: Again using the design report inlet conditions  $\rho_{inlet}$  and  $s_{inlet}$  are found, and  $h_{inlet}$  is found to be 166.9 J/g.

For example, for the cutoff = 0.46 as above  $\rho_{expanded} = 1.434 \times 10^{-2} \text{ g/cm}^3$ . For this density and  $s = 13.03 \text{ J/g}^\circ\text{K}$  the NBS Helium tables indicate  $h = 101.0 \text{ J/g}$ . Thus  $\Delta h = 166.9 - 101.0 = 65.9 \text{ J/g}$ .

Wet engines: The same procedure described above is followed using either the NBS Helium tables or a T-S diagram.

For both Wet and Dry engines the thermodynamic efficiency is defined as the real change in enthalpy (generally found from inlet and exhaust temperatures and pressures) divided by the ideal (isentropic) change in enthalpy. For consistency and convenience the isentropic expansion in calculating efficiency is assumed to be from 20 atmospheres to 1.2 atm for the Dry engine, and to 1.8 atm for the Wet.

Hence for the Dry engine the expansion in the cylinder to a pressure greater than 1.2 atm is one possible source of inefficiency, a trade off made in order to obtain a greater flow rate for a lower engine speed.

For example, if a Gardner-Fermi Dry engine were operating at 297 RPM with a  $\Delta h$  (estimated from inlet and exhaust temperatures and pressures ) of 70 J/g and a cutoff (estimated from the oscilloscope trace of cylinder pressure) of 0.33 the calculated efficiency would be  $70 \text{ J/g} / 105 \text{ J/g} = 0.67$ . But actual expansion in the cylinder would be to 2.8 atm followed by throttling to 1.2 atm. This expansion relative to the isentropic at 2.8 atm is  $70 \text{ J/g} / 84 \text{ J/g} = 0.83$ . Although this indicates that the engine would be considered more efficient if it were exhausting to 2.8 atm and 2.8 atm were used as the reference point in calculating efficiency, it does not mean that with a 0.21 cutoff (a different inlet cam) the engine would operate at 83% efficiency expanding to 1.2 atm. Randall Barron, in "Cryogenic Systems" (McGraw-Hill, 1966, page 168) states that "the early cutoff is sometimes advantageous because a relatively small amount of work is produced by the final 10 percent of the piston stroke, whereas friction and heat transfer losses are highest in this region. By shortening the cutoff, the friction losses are reduced at the expense of the small amount of work output at the end of the stroke". More data are needed in order to see where the practical optimum is for the cutoff on our Dry engines.

#### Column 7

The design report calls for  $57\frac{1}{2} \text{ g/sec}$  total flow, so  $30 \text{ g/sec}$  through the Dry engine is about  $\frac{1}{2}$  total flow, such as during cooldown, and  $20 \text{ g/sec}$  is about  $\frac{1}{3}$  total flow, such as in refrigerator mode.

The Wet engine in turn would see as much as 60 g/sec during satellite mode (Dry engine turned off) or 40 g/sec during refrigerator mode.

Column 8

$$\text{Speed (RPM)} = \text{flow (g/sec)} \times \frac{60 \text{ sec}}{\text{min}} \times \frac{\rho_{\text{inlet}} \frac{q}{\text{cm}^3} \times \frac{\text{inlet volume (cm}^3\text{)}}{\text{revolution}}}{1}$$

where inlet volume is total volume swept by piston times the intake cutoff.

Speeds in parentheses are higher than the recommended maxima, which are 450 RPM for the Gardner-Fermi (limited by the electric drive and present pulley ratio, safety brake set to trip at about 600 RPM) and 300 RPM for the CTI engines (per CTI recommendation, actually limited to about 300 RPM by electric drive and present pulley ratio, brake set to trip at about 400 RPM).

Column 9

$$\begin{aligned} \text{Ideal power out (KW)} &= \\ &= \Delta h_{\text{ideal}} \text{ (j/g)} \times \text{flow (g/sec)} \times \frac{1 \text{ KW}}{1000 \text{ W}} \end{aligned}$$

We can read the power output of the DC drive while the engine is running, and typical values for CTI-D-3 at B1 have been  $0.6 \pm 0.4$  KW. This observation seems to confirm the experience of George Mulholland (verbally described to me) that such measurements will be low. He attributes this to various mechanical losses, such as frictional heating of the crosshead guides, bearings, and belts and pulleys, and I would also add the inefficiency of the DC drive as a generator. Average mechanical efficiency (defined as indicated power out on the DC drive divided by real power removed from the gas as estimated from measured temperatures, pressures, and engine speed) for CTI-D-3 at B1 has been about 0.67.

Perhaps with more data we can still correlate this reading with flow rate times real  $\Delta h$ . We could then use RPM's and power out as a quick check of efficiency at temperatures outside of the VPT ranges.



However, this task is complicated by the fact that flow rate depends on density as well as RPM's, which in turn depends on temperature and pressure.

Column 10

The heat input per gram of gas is  $\frac{\text{heat (W)}}{\text{flow (g/sec)}}$   
 = enthalpy change (J/g), and the % effect is taken relative to  $\Delta h_{\text{ideal}}$  (column 6).

CTI engineers have estimated the heat leak to be 16W per cylinder in CTI's engines. Hence the 32W nominal figure.

Column 11

"Blowby" generally refers to gas which leaks past the cylinder rings to the low pressure side and hence does no work. Since the CTI expanders have only warm rings sealing to the atmosphere, this kind of "blowby" which essentially throttles the helium to the low pressure stream only can occur in the Gardner-Fermi expander. However, the effect of a leaking exhaust valve is the same, hence entries are included for the CTI expanders.

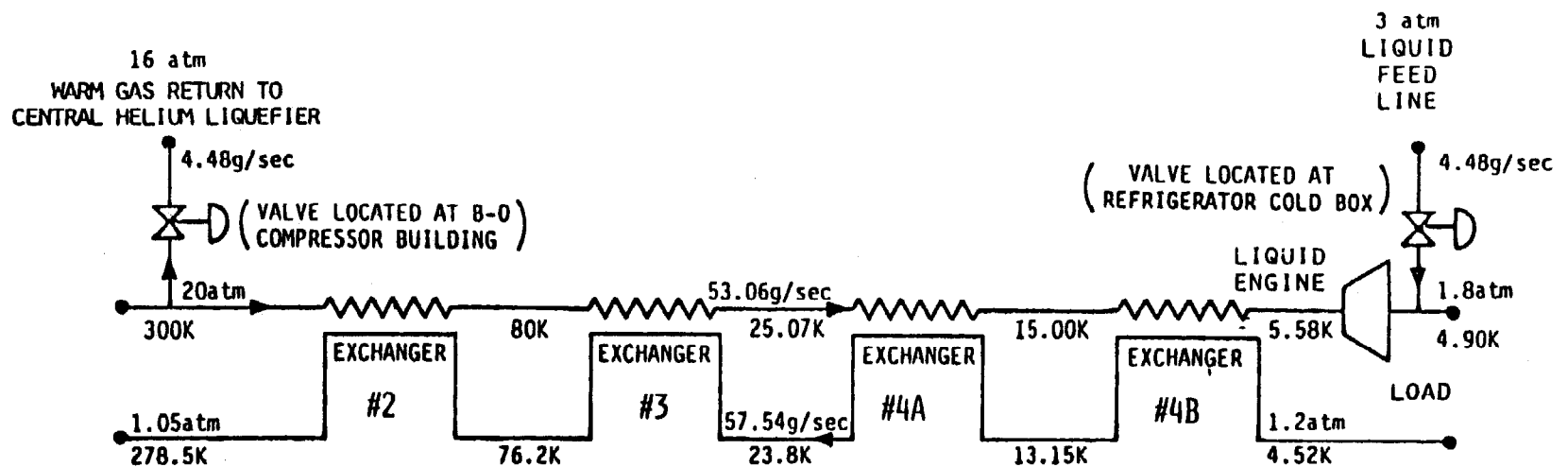
$$\frac{\Delta h_{\text{ideal}} (\dot{m}_{\text{total}} - \dot{m}_{\text{blowby}})}{\dot{m}_{\text{total}}} = \Delta h_{\text{net}}, \text{ so reduction due to blowby}$$

of  $\Delta h$  is  $\Delta h_{\text{ideal}} - \Delta h_{\text{net}} = \Delta h_{\text{ideal}} \left( \frac{\dot{m}_{\text{blowby}}}{\dot{m}_{\text{total}}} \right)$ .

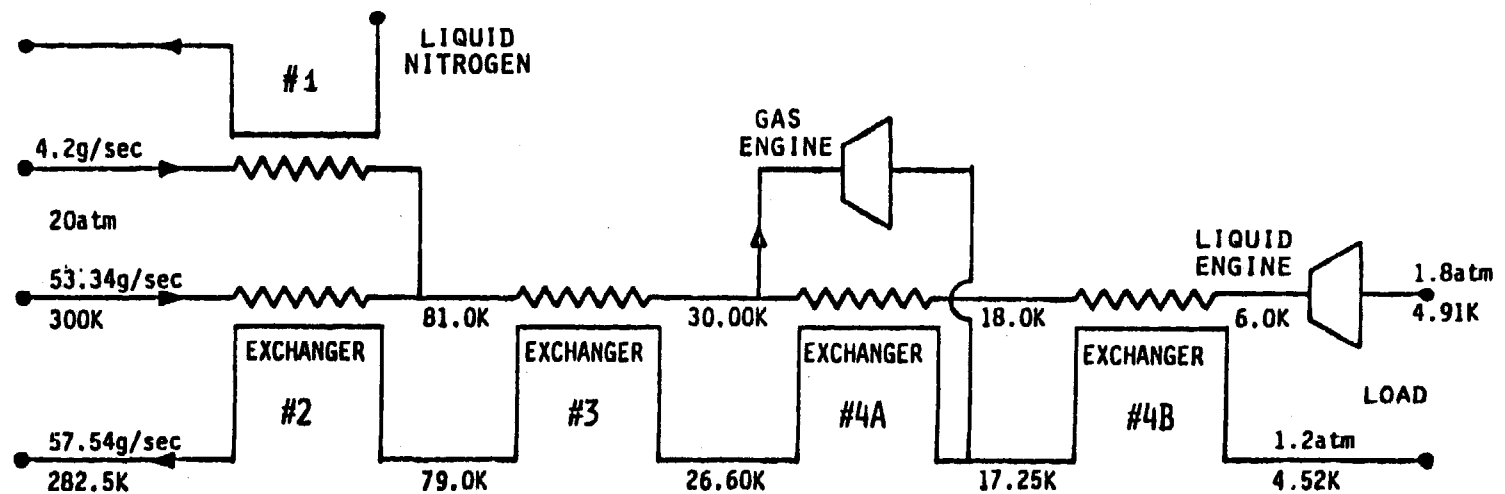
As in the case of the heat leak, the % effect is taken relative to  $\Delta h_{\text{ideal}}$  (column 6).

Conclusion.

Calculations like these are at least as useful for the questions they raise as for the answers they provide. Among the important questions apparent here which experience has to answer are what Dry engine intake cutoff is "best" (presumably by some overall refrigeration criteria), what collet clearance (in the case of the CTI engines) will give us the desired effective cam sizes, and can the "Power Out" reading on our controllers give us a quick check on engine performance.



### SATELLITE MODE



### REFRIGERATOR, LIQUEFIER & E.D. STANDBY MODE (refrigerator values illustrated)

Fig. 4-4. Satellite refrigerator modes.

FIGURE 1

TABLE OF EXPANDER  
CALCULATIONS

1	2	3	4	5	6	7	8	9	10	11
Type of Engine	Intake Cam Size	Intake Cutoff	Inlet Temp Assumed ( $^{\circ}\text{K}$ )	Cylinder Pressure after Ideal Expansion (atm)	$\Delta h$ Ideal for Expansion to Theoretical Final Cylinder Pressure ( $\text{J/g}$ )	Flow Rate ( $\text{g/sec}$ )	Engine Speed (RPM)	Ideal Power Out (Using Column 6) (kW)	Reduction of $\Delta h$ (and % Effect) of 32 W Heat Leak ( $\text{J/g}$ )	Reduction of $\Delta h$ Due to $\frac{1}{2}$ g/sec Blowby ( $\text{J/g}$ )
Gardner-Whermi Dry 3.187" Dia piston x 3.00" stroke 1 piston	100 $^{\circ}$ 15 $^{\circ}$ "lost"	0.46	30	5	66	20	213	1.32	1.6 (2%)	1.7 (2½%)
						30	320	1.98	1.1 (2%)	1.1 (2%)
	85 $^{\circ}$ 15 $^{\circ}$ "lost"	0.33	30	2.8	84	20	297	1.68	1.6 (2%)	2.1 (2½%)
						30	446	2.52	1.1 (1%)	1.4 (2%)
	70 $^{\circ}$ 15 $^{\circ}$ "lost"	0.21	30	1.2	105	20	467	2.10	1.6 (2%)	2.6 (2½%)
						30	(701)	3.15	1.1 (1%)	1.8 (2%)
	90 $^{\circ}$ 8 mil collet clearance	0.46	30	5	66	20	181	1.32	1.6 (2%)	1.7 (2½%)
						30	271	1.98	1.1 (2%)	1.1 (2%)
CTI Dry 3.00" Dia piston x 2.00" stroke 2 pistons in parallel	90 $^{\circ}$ 20 mil collet clearance	0.38	30	3.5	79	20	219	1.58	1.6 (2%)	2.0 (2½%)
						30	(329)	2.37	1.1 (1%)	1.3 (2%)
	90 $^{\circ}$ 35 mil collet clearance	0.33	30	2.8	84	20	252	1.68	1.6 (2%)	2.1 (2½%)
						30	(378)	2.52	1.1 (1%)	1.4 (2%)
	120 $^{\circ}$ 10 mil collet clearance	0.70	8	2.5	14.9	40	125	0.60	0.8 (5%)	0.2 (1.3%)
						60	188	0.89	0.5 (4%)	0.1 (0.8%)
CTI Wet 2.00" Dia piston x 2.00" stroke 2 pistons in parallel	120 $^{\circ}$ 10 mil collet clearance	0.70	6	1.2 (approx)	13 (approx)	40	110	0.52	0.8 (6%)	0.2 (1.3%)
						60	165	0.78	0.5 (4%)	0.1 (0.8%)
	120 $^{\circ}$ 10 mil collet clearance	0.70	5.6	0.8 (approx)	13 (approx)	40	108	0.52	0.8 (6%)	0.2 (1.3%)
						60	162	0.78	0.5 (4%)	0.1 (0.8%)
	120 $^{\circ}$ 10 mil collet clearance	0.70	5.6	0.8 (approx)	13 (approx)	40	108	0.52	0.8 (6%)	0.2 (1.3%)
						60	162	0.78	0.5 (4%)	0.1 (0.8%)

FIGURE 2

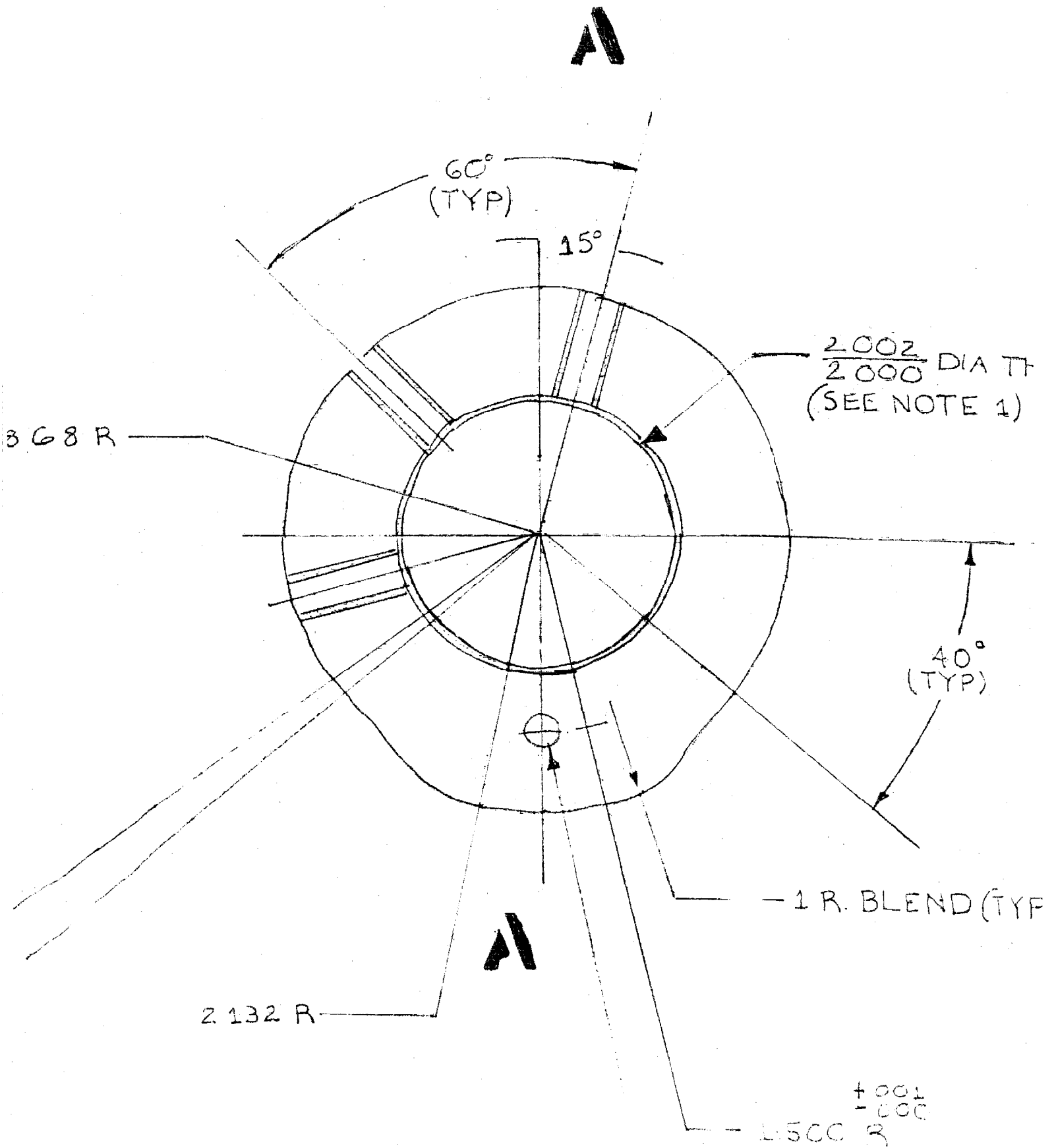


FIGURE 5

Effect of Tappet Clearance on  
Intake Cutoff for Gardner-Fermi  
Dry Engine (100° cam)

Tappet Clearance (inches)	Cam Advancement (degrees)	Intake cutoff
0.005	8	0.46
0.010	8	0.44
0.020	8	0.41
0.030	8	0.38
0.040	8	0.37

Lift at collet (mils.)

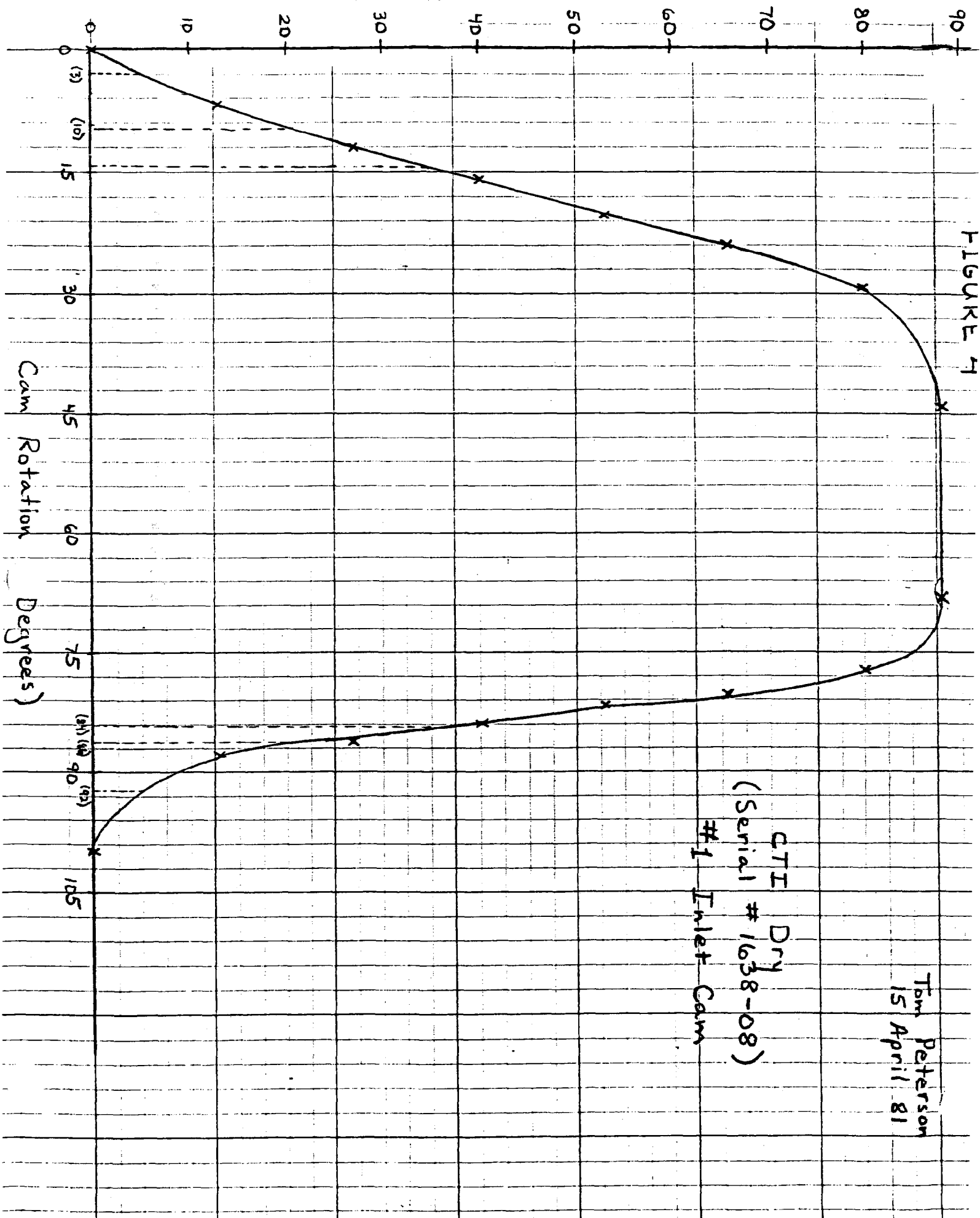


FIGURE 5

Effect of Collet Clearance on  
Intake Cutoff for CTI  
Dry Engine

Collet Clearance (inches)	Cam Advancement (degrees)	Intake Cutoff
0.005	10	0.43
0.010	10	0.40
0.020	10	0.38
0.035	10	0.36
0.008	4	0.46
0.035	14	0.33



FIGURE 6

